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High-Altitude Explosions and Their Magnetohydrodynamic Description

Yuri P. Raizer* and Sergei T. Surzhikov†
Russian Academy of Sciences, 117526 Moscow, Russia

The paper provides approaches for formulating general ideas of a gas dynamic stage for an explosion in the upper atmosphere in which the geomagnetic field is of importance. Various versions concerned with magnetic pressure influence, particle collisions, and the latitudinal atmosphere nonuniformity are discussed. The following magnetogasdynamic models are constructed: 1) a quasitwo-dimensional model which treats the spherically symmetric flows with the magnetic force averaged over angles, 2) a two-dimensional model, and 3) a three-dimensional model. The latter model is applied to a general case of an arbitrary directed magnetic field. The computational results obtained for particular flows according to the magnetohydrodynamic (MHD) models are given. In the next paper we substantiate the MHD-theory application even for very rarefied magnetized plasma.

Nomenclature

E	= kinetic energy of the expanding explosion products
E_t	= total explosion energy
f	= averaged ponderomotive force
H	= magnetic field
H_0	= undisturbed geomagnetic field
h	= altitude
ℓ	= particle mean free path
L	= typical plasma flow scale
M	= explosion product mass
M_A	= Alfven-Mach number
n	= particle density
n_e	= electron density
p	= pressure
p_m	= magnetic pressure
R	= blast wave radius
R_g	= hydrodynamical retardation radius
R_m	= magnetic retardation radius
r	= radial coordinate
t	= time
u	= flow velocity
V_A	= Alfven velocity
γ	= adiabate exponent
Δ	= atmospheric nonuniformity scale
θ	= polar angle
ρ	= gas density
ρ_0	= atmospheric density at the explosion altitude
σ	= plasma conductivity

Introduction

THE experimental nuclear explosions like Argus (with energy E_t equivalent to 10^3 ton = 1 kt of TNT = 4.2×10^{19} erg and altitude $h = 500$ km) in 1958[1] and Starfish ($E_t \cong 1.4$ Mt, $h = 400$ km) in 1962[2] give rise to many very interesting physical problems. Some of them are of general scientific significance, since similar magnetohydrodynamic (MHD) processes occur in such space phenomena as the solar wind and shock waves in the Earth's magnetosphere. We encounter these problems in space experiments[3–5] when a small amount of energetic plasma is injected into the ionosphere, and in some related laboratory experiments.[6] The practical importance is confirmed, in particular, by that serious damage to the communication links and the components of a space defense system which might be caused by a high-altitude nuclear explosion.[7]

The very first problem, without which it would be difficult to consider any other problems, is that of the expansion of the explosion product plasma into the very rarefied ambient atmosphere with the geomagnetic field $H_0 \cong 0.5$ Oe, the explosion being accompanied by the explosion product (EP) deceleration due to the medium resistance and magnetic field action, by blast wave propagation in the atmosphere, and by magnetic field distortion. This stage of the process, up to the strong wave decay and the fast motion deceleration, can last up to tens of seconds and covers hundreds of kilometers. (We call this stage a gasdynamic one and the paper is devoted to its consideration.) The analysis of the gasdynamic stage should show how the electron and radioactive gas atom densities and the magnetic field are distributed in space and how these distributions evolve. Note only are the blast actions on space objects and communication links determined by this stage, but the gasdynamic stage also provides the initial conditions for the subsequent relatively slow long duration processes. The perturbations of the ionosphere and radiation belt properties due to the Argus and Starfish explosions were registered for many hours.

Range of Gasdynamic Parameters and Theoretical Model Choice

The gasdynamic processes and, hence, an adequate theoretical model to describe them depend essentially on the explosion altitude and energy; but even for h and E_t fixed they are often different for different points of the blast wave. The situation depends on the relationship between magnetic ($p_m = H_0^2/8\pi = 10^{-2}$ dyne/cm^2) and atmospheric pressure p_0 ($p_m \cong p_0$ at $h = 140$ km) and on the relationship between a typical EP expansion radius (or blast wave radius $R = 100$–500 km) and the particle mean free path ℓ. Both of these ratios vary with the altitude. Also of influence is the relationship between R and a height nonuniformity scale Δ of the atmosphere. In the vertical direction the undisturbed air density varies e times along a distance Δ; at sea level $\Delta \cong 8$ km, whereas at $h = 150$ km, we have $\Delta \cong 50$ km.

The mean free paths of particles in a wide and practically important altitude range ($h \cong 200$–700 km) appear to be comparable (at certain times) with the size of a region involved in the motion. This is the most difficult complication of the theory; furthermore, these paths are different for different particles. For example, ions of EP expanding at velocities $u_0 \cong 10^2$–10^3 km/s and neutral atoms of the medium will have significantly different mean free paths. Formally, for $\ell > R$ one cannot use the continuum mechanics equations. The question arises regarding which are the possible mechanisms involving the ambient medium in the motion when the collisions

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*Dr. Sci., Professor, Head, Department of Physical Gasdynamics, Institute for Problems in Mechanics, 101 Vernadsci St.

†Dr. Sci., Leading Researcher, Department of Physical Gasdynamics, Institute for Problems in Mechanics, 101 Vernadsci St.

are extremely rare. The charged particles in the ionosphere can be accelerated by the electromagnetic interactions in the external magnetic field. Neutral atoms can begin to move only after ionization or charge transfer. One more question that arises is whether there is a collisionless mechanism to heat electrons, since collisionally heated electrons do not have sufficient energy to ionize atoms. Perhaps, the plasma collective interactions that lead to the ion kinetic energy dissipation by the plasma oscillation growth provide a possible collisionless mechanism of electron heating.

One can avoid many of these complications by considering two limiting, or idealized, situations. They are not only of importance and of interest by themselves, but also serve as reference points in treating the already discussed, more complicated cases.

1) Considering other than high-intensity explosions ($E_t \le 10^2$ kt) at not too high altitudes, say, less than 100 km, we see that $\ell \ll R$, $p_m \ll p_0$. In this situation, the process can be considered using the standard gasdynamic equations, with the magnetic field neglected.

Sometimes it is necessary to take into account nonequilibrium gas phenomena such as ionization kinetics, the difference between electron and gas temperatures, and radiation heat exchange and radiation energy losses, but these difficulties, in contrast to those already mentioned, are not a matter of principle. In general, such situations were considered in Ref. 8 (see also Ref. 9).

2) In the case of not too energetic explosions at very high altitudes, $h > 500$ km, when $p_m \gg p_0$, the EP plasma cloud is decelerated by magnetic pressure (except perhaps that portion which moves vertically downwards). The expansion ceases at a distance of magnetic retardation R_m which can be estimated when equating kinetic and magnetic pressures[10]

$$3E/4\pi R_m^3 = p_m = H_0^2/8\pi,$$

$$R_m = (3E/4\pi p_m)^{\frac{1}{3}} = (6E/H_0)^{\frac{1}{3}} \tag{1}$$

In the case of a highly energetic explosion, e.g., $E_t \simeq 1$ Mt, about 80% of the energy is carried far away during the first time moments by the thermal rays, so that $E \simeq 0.2E_t$. For low-energy explosions ($E_t \simeq 1$ kt) the energy radiation is small, and almost all explosion energy is transformed into EP expansion energy ($E \simeq E_t$).

Let us introduce the radius R_g of a sphere around the explosion point, with the sphere containing a medium mass equal to the EP mass

$$M = \rho_0\left(4\pi R_g^3/3\right), \qquad R_g = (3M/4\pi\rho_0)^{\frac{1}{3}} \tag{2}$$

One can think of R_g as a characteristic radius of hydrodynamic EP retardation. If the medium covered by the cloud were fully involved in the motion, then the EP kinetic energy would decrease by half, and the cloud expansion velocity would decrease by a factor of $\sqrt{2}$ compared with the initial mean expansion velocity $u_0 = \sqrt{2E/M}$.

Let us assume

$$R_m/R_g = (u_0/V_A)^{\frac{2}{3}} = M_A^{\frac{2}{3}} < 1 \tag{3}$$

where M_A is an effective Mach number based upon the Alfven velocity V_A. In this case only a small amount of the medium mass (essentially less than the EP mass) can join the cloud before it stops. This means that the cloud expands as if there were no ambient medium at all. For instance, for the Argus explosion we have $E \simeq$ 1 kt, $M \cong 1$ t, $u_0 \simeq 90$ km/s, $R_m \cong 100$ km. At the altitude $h = 500$ km we have $\rho_0 = 5 \times 10^{-16}$ g/cm^3, $R_g = 80$ km, and $M_A = 1.4$. The atom mean free path in the atmosphere (here the degree of dissociation is very high) $\ell \cong 80$ km, and that of the fast EP ions is even higher. One can approximately consider the explosion to occur in a vacuum. This situation was considered in Refs. 10 and 11. As is shown in Ref. 10, the EP cloud displacing the magnetic field loses an essential part of its energy before it stops. The energy is carried away to infinity by the electromagnetic impulse together with part of the displaced field energy. After the main part of the cloud stops ($t \simeq 1$ s), the charged particles appear to be trapped inside a magnetic cavity, i.e., they are reflected by its walls. Within this cavity, their density is of the order of $n \simeq 10^7$ cm^{-3} which is much greater than the ionospheric density. Eventually, particles gradually run away through a throat of the magnetic bottle along the magnetic field lines (the magnetic field structure is similar to the one, shown in Fig. 2). Two-dimensional calculations[12] confirm that after the cloud stops its boundary begins to oscillate.

The aim of the present paper is to suggest approaches for formulating general ideas on the gasdynamical stage of an explosion. The formulation should enable estimates to be made of the duration, spatial scales, and distributions of perturbations in those complicated intermediate situations which take place between previously considered, simpler limiting cases. In general, to formulate such ideas we use the MHD models. Application of these models under the conditions of $\ell > R$ can be justified when comparing with our computations according to the hybrid model[13] in which the collisionless plasma motion in a magnetic field is considered in the frame of the kinetic equation for ions and hydrodynamical equations for electrons. As will be shown in our next paper,[13] the MHD model gives reasonable results even for the collisionless case where it is formally invalid.

Magnetohydrodynamic Description

The description is based on the gasdynamics equations, including additional terms which take into account the magnetic field and energy release due to electric currents. The approximation of the perfect conductivity for the medium is often used, $\sigma = \infty$. The magnetic field is determined by the equation

$$\frac{\partial \boldsymbol{H}}{\partial t} = \mathrm{rot}[\boldsymbol{u} \times \boldsymbol{H}] \tag{4}$$

which follows from Maxwell's equations if the displacement currents are neglected (which is valid for the nonrelativistic motion) and the conductor is considered to be perfect. The last assumption needs to be clarified.[14]

Since $\sigma = e^2 n_e/m\nu_m$, one can pass to the limit $\sigma = \infty$ in two ways, namely, as $\nu_m \Rightarrow 0$ and as $n_e \Rightarrow \infty$. Equation (4) is obtained from the more general one

$$\frac{\partial \boldsymbol{H}}{\partial t} = \mathrm{rot}[\boldsymbol{u} \times \boldsymbol{H}] - \frac{c}{(4\pi e)}\mathrm{rot}\{n^{-1}[\mathrm{rot}\,\boldsymbol{H} \times \boldsymbol{H}]\}$$

$$- \,\mathrm{rot}\left\{\frac{c^2}{(4\pi\sigma)}\mathrm{rot}\,\boldsymbol{H}\right\} \tag{5}$$

if we omit the second and third terms on the right-hand side. This is possible when the inequalities

$$æ = cH/(4\pi enuL) \ll 1, \qquad c^2/4\pi\sigma uL \ll 1 \tag{6}$$

are satisfied. These two dimensionless parameters characterize the ratio of the omitted terms to that which remains. The second requirement, $\sigma = \infty$ (which corresponds to the case when one can neglect diffusion, field penetration into plasma and Joule heating), is not sufficient. It is also necessary for the electron density to be rather high ($n = \infty$) independently of the magnitudes of ν_m and σ. At altitudes higher than $h \cong 100$ km, the first requirement of Eq. (6) is more restrictive. Indeed, the third to second term ratio in Eq. (5) is ν_m/Ω_e, where $\Omega_e = eH/mc$ is equal to 0.9×10^7 s^{-1} in the Earth field. For $h > 100$ km we have $\nu_m < \Omega_e$, i.e., the second term in Eq. (5) is greater than the third one. However, even the natural ionization in the ionosphere is almost sufficient to satisfy the inequality $æ < 1$; hence, the restriction to apply a gasdynamic model, $\ell < L$, is more serious than Eq. (6) ($n = \infty$, $\sigma = \infty$).

There is a rather wide and practically interesting range of altitudes ($h \simeq 150$–500 km) where $p_0 \ll p_m$, but for which the mean free paths of the particles are still not too large, so that $\ell \le L \simeq 100$ km. (For the atmospheric particles $\ell \simeq 80$ km for $h \cong 500$ km). Therefore, we can expect to obtain reasonable results when using the MHD model in the aforementioned range, at least, to describe the horizontal and downward motions. As will be shown in Ref. 13, one can expect to obtain appropriate results even when considering the upward motion at h, higher than 500 km, even though there we have $\ell > L$.

Explosion in the Uniform Atmosphere Treated According to Quasitwo-Dimensional Magnetohydrodynamic Model

In a general case the large-scale motion in the upper atmosphere is three dimensional since there are two special directions, the vertical one and the H_0 direction. For not very high altitudes and energies for which the shock wave decays at a distance which is not essentially greater than Δ (which for the upper layers of the atmosphere is about 50 km), the motion occurs in almost uniform medium and is two dimensional. To describe such a situation as well as the motion in the nonuniform atmosphere along a certain direction (for example, upwards), a simple quasitwo-dimensional MHD model is suggested.[14] By averaging over angles, the real two-dimensional problem can be reduced to a one-dimensional one which essentially diminishes the computer time and makes it possible to simplify computations.

Let us use the polar coordinates r and θ. The medium is considered to be uniform with H_0 as the symmetry axis. Neglecting the gas flow across an angle, we consider the motion to have spherical symmetry. The MHD equations are of the form

$$\frac{\partial\rho}{\partial t} + \frac{r^{-2}\partial(r^2\rho u)}{\partial r} = 0$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r}\right) = -\frac{\partial p}{\partial r} + \bar{f}_r \qquad (7)$$

$$\frac{\partial[\rho(\varepsilon + u^2/2)]}{\partial t} + \frac{r^{-2}\partial[r^2\rho u(\varepsilon + p/\rho + u^2/2)]}{\partial r} = \bar{f}_r u$$

$$\varepsilon = p/(\gamma - 1)\rho$$

When considering an explosion in the uniform atmosphere, we substitute the radial component of the magnetic force

$$f_r = (4\pi)^{-1}[\mathrm{rot}\,\boldsymbol{H} \times \boldsymbol{H}]_r = \frac{H_\theta}{(4\pi r)}\left[\frac{\partial H_r}{\partial\theta} - \frac{\partial(rH_\theta)}{\partial r}\right] \qquad (8)$$

In Eq. (7) we use an average value of f_r over solid angles, which yields $\bar{f}_r$. Considering the motion in a certain direction, let us assume the undisturbed medium density to be dependent on r (taking into account the atmosphere's nonuniformity). We replace $\bar{f}_r$ by the appropriate function $f_r(r, \theta)$ according to Eq. (8). In this approximation the velocity $u = u_r$ does not depend on θ, and it is possible to separate variables

$$H_r = H_r^0(t, r)\cos\theta, \qquad H_\theta = H_\theta^0 \sin\theta \qquad (9)$$

in the equations for H_r and H_θ

$$\frac{\partial H_r}{\partial t} + \left(\frac{u}{r^2}\right)\frac{\partial(r^2 H_r)}{\partial r} = 0$$

$$\frac{\partial H_\theta}{\partial t} + \frac{r^{-1}\partial(ruH_\theta)}{\partial r} = 0 \qquad (10)$$

following from Eq. (4). The new functions H_r^0 and H_θ^0 satisfy the same Eq. (10). They are related by the formula

$$H_\theta^0 = -\left(\frac{1}{2r}\right)\frac{\partial(r^2 H_r^0)}{\partial r}$$

which follows from the equation $\mathrm{div}\boldsymbol{H} = 0$ and is not independent.

The radial force f_r inducing the magnetic deceleration falls off, as $f_r \sim \sin^2\theta$, from its maximal value for $\theta = \pi/2$ down to $f_r = 0$ for $\theta = 0, \pi$. This leads to the distortion of the initially spherical blast wave; on the wave front the throats are formed elongated in the H direction. The force averaged over a sphere is

$$\bar{f}_r = -(2/3)(H_\theta^0/4\pi)\partial(H_\theta^0 - H_r^0/2)/\partial r$$
$$= \left(\frac{2}{3}\frac{\partial(H_\theta^0)^2}{8\pi}\right) + \left(\frac{2}{3}\right)\frac{(H_\theta^0/8\pi)\partial H_r^0}{\partial r} \qquad (11)$$

The first term, corresponding to the H_θ component transverse to the motion direction, can be treated as magnetic pressure action $(H_\theta^0)^2/12\pi$ averaged over angles.

As an example, the computational results for the explosion of $E = 3 \times 10^{20}$ erg $\simeq 7$ kt in a uniform atmosphere with $\rho_0 = 4.3 \times 10^{-13}$ g/cm^3, $p_0 = 1.8 \times 10^{-3}$ dyne/cm^2 (corresponding to $h \simeq 180$ km; $\gamma = 5/3$, $H_0 = 0.5$ Oe) are given in Figs. 1 and 2. Maximal (at the equator) magnetic back pressure p_m is 5.5 times more than the atmospheric pressure. This shows that when the shock is decaying into a weak disturbance, the magnetic deceleration dominates. A great amount of the air mass is involved in the motion, 6×10^3 t at the time moment $t = 20$ s when the shock wave covering a distance of $R = 150$ km becomes weak.

At the earlier stages, the p, ρ, and u distributions do not differ from self-similar distributions obtained when solving the explosion problem without the magnetic field.

At the later stage, the wave gradually degenerates into a weak decaying disturbance which propagates at a velocity close to the Alfvenian, $V_A = 1.5$ km/s. Behind the wave the motion gradually decays. The final pressure (equalized in the central region) tends to $p_m > p_0$. This situation differs from the nonmagnetic case, in which the atmosphere pressure p_0 is restored as $t \Rightarrow \infty$. The magnetic field component H_θ (transverse to the motion direction) remains proportional to the gas density, in accordance with the assumption of the frozen magnetic field.

About 25% of the energy E is transformed into the magnetic energy toward the end of the process. About 30% of the energy remains in the central empty sphere with a radius of 120 km, due to irreversible heating by the shock wave. (The situation is similar to that in the lower layers of the atmosphere where the energy is radiated from a fire ball.[8]) The remaining 45% of the energy is carried away by the decaying wave. The magnetic field force lines take the typical form shown in Fig. 2. They are displaced from the volume by plasma, the displacement being greater in the equatorial direction and weaker along the directions close the polar one, i.e., to H_0. But the plasma body itself in the quasitwo-dimensional approximation retains the spherical symmetry.

On solving the gasdynamical problem, we obtain the expansion and cooling of a gas mass element after their heating by a strong shock wave. Considering the subsequent ionization kinetics, one can find the electron density n_e in the mass element as a function of time. Such data for all particles give the n_e distribution in space and time.

Upwards Propagation of the Explosion Blast Wave Described by a Quasitwo-Dimensional Magnetohydrodynamic Model

When considering an explosion in the nonuniform atmosphere, it is reasonable to choose a simple standard model of the atmosphere. To take into account the total composition of the atmospheric gas, one can set (within the accuracy sufficient for the case)

$$\rho = 10^{-12}\exp[-(h - 150)/\Delta]\ \mathrm{g/cm^3}$$
$$\Delta = 42.2\ \mathrm{km}, \qquad h \geq 150\ \mathrm{km}$$
$$p = 4.5 \times 10^{-3}\exp[-(h - 150)/\Delta]\ \mathrm{dyne/cm^2} \qquad (12)$$

For computations according to this model, the atmospheric gas is assumed to be fully involved into the motion in a manner which is independent of the degree of rarefaction. If we assume that at high altitudes where there are no collisions only charged particles of the ionosphere can be involved in the motion, in which case

$$\rho = 10^{-12}\exp[-(h - 150)/\Delta_1]\ \mathrm{g/cm^3}$$
$$\Delta_1 = 36.6\ \mathrm{km}, \qquad 150 \leq h < 500\ \mathrm{km}$$
$$\rho(h) = \rho(500) = 1.22 \times 10^{-15}\ \mathrm{g/cm^3}, \qquad h > 500\ \mathrm{km} \quad (13)$$

The quantity Δ_1 is chosen to be slightly different from Δ to provide a smooth transition from the atmosphere gas density at $h = 150$–300 km to the density of ionospheric charged particles at $h \geq 500$ km.

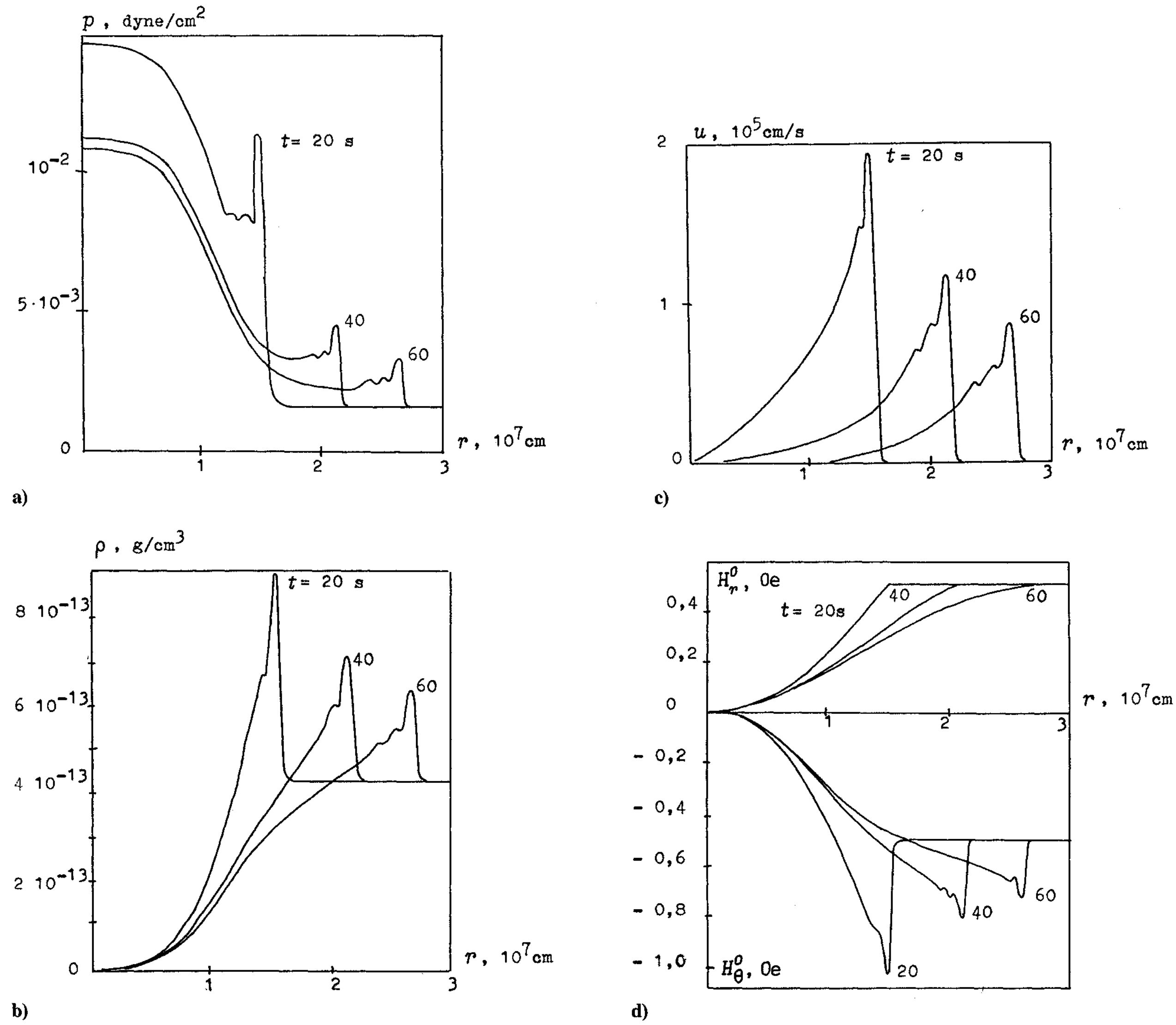

Fig. 1 Explosion of energy $E = 3 \times 10^{20}$ erg, uniform atmosphere, initial magnetic pressure 5.5 times atmospheric pressure, quasitwo-dimensional MHD-model computations for times $t = 20$ s, $t = 40$ s, and $t = 60$ s: a) radial distributions of gas pressure, b) density, c) velocity, and d) axial magnetic field at the magnetic poles (H_r^0) and transverse to the radial field (H_θ^0) on the magnetic equator.

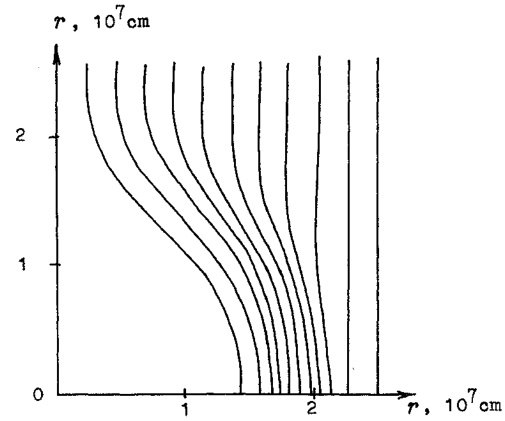

Fig. 2 Magnetic field lines based on Fig. 1d and Eqs. (10) for $t = 40$ s.

In Fig. 3 the numerical results are shown for a quasitwo-dimensional explosion in an atmosphere of type (12), with the same density ρ_0 at the explosion point and the same magnetic force f_r as in previous computations. The solution describes the upwards blast wave propagation for an inclination of H_0 to the vertical at about $\arcsin\sqrt{2/3} \approx 54$ deg, such as might occur at middle latitudes. This is just that angle between H_0 and the vertical for which magnetic pressure coincides with the angle-averaged magnetic pressure used in the computations. Since according to Eq. (12) the density falls off down to zero, then as in the case without a magnetic field, the wave front expands to infinity for a finite time.[8] (Note that the calculations shown in Fig. 3 were terminated before this time.) But now the cause of unlimited shock acceleration is different. Without the field,the shock wave is accelerated due to its amplitude growth, i.e., due to the pressure ratio at the shock front induced by energy accumulation.[8] On the contrary, when there is a constant magnetic back pressure p_m the shock wave amplitude falls off, and the shock is gradually degenerated into a weak disturbance. But the velocity of the disturbance (which is equal to the Alfvenian one) grows because the Alfvenian velocity grows as $1/\sqrt{\rho}$.

Two-Dimensional Magnetohydrodynamic Model

The model is based on the axially symmetrical flow equations written in the cylindrical coordinates r, z, with the axis z being directed along H_0. The initial atmospheric density can vary along the z axis. The two-dimensional computations confirm that the general results obtained on the basis of the simpler quasitwo-dimensional model are correct, and the model can be accepted for the serial computations when special details are of no interest. To illustrate the results obtained when using the two-dimensional model, the

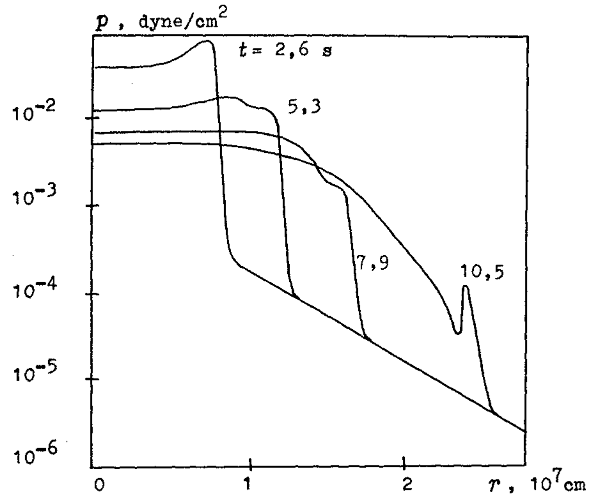

Fig. 3 Pressure distributions along the vertical for various times using MHD-model computations for an explosion in the atmosphere with exponentially decreasing density; energy and medium density at the explosion point are the same as Figs. 1 and 2.

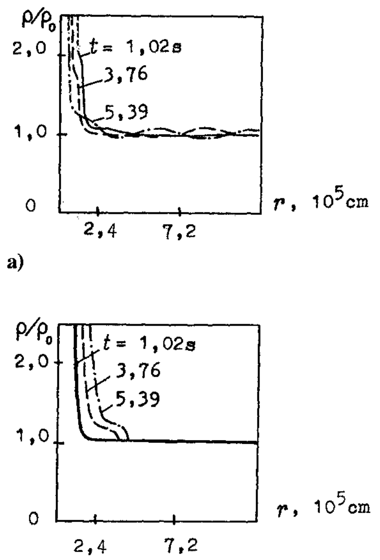

Fig. 4 Computational results of the gas motion in the atmosphere at 190-km altitude, motion induced by the plasma cloud expansion (14.8 kg), energy equal to 3×10^{13} erg (as in the "Dogwood" experiment) at time $t = 1.02$ s (1), $t = 3.76$ s (2), and $t = 5.39$ s (3), computed with cylindrical coordinates, z axis directed along the undisturbed magnetic field: a) density distributions along the radius in the diametrical plane passing through the center and b) along the x axis. The density is normalized by the atmospheric density ρ_0, whereas the distances by the hydrodynamic retardation radius $R_g = 1.79$ km.

computational data are given for the real two-dimensional motion in the Dogwood experiment of Ref. 3 (Figs. 4–6).

In this experiment ($h = 190$ km) a barium plasma ($M = 14.8$ kg) was injected into the atmosphere [$E = 3 \times 10^{13}$ erg, corresponding to a mean initial expansion velocity $u_0 = (2E/M)^{\frac{1}{2}} = 0.63$ km/s]. The magnetic retardation radius R_m is 0.8 km $\ll \Delta$. The deceleration due to magnetic pressure $p_m = 5.5 p_0$ ($p_0 = 1.8 \times 10^{-3}$ dyne/cm^2 and $\rho_0 = 4.3 \times 10^{-13}$ g/cm^3) is greater than that due to atmospheric pressure. The hydrodynamic retardation radius R_g is equal to 2 km $\geq R_m$. The molecule mean free path in the air is $\ell \cong 180$ m, which is several time less than both R_m and R_g, i.e., the motion is not collisionless. At time $t \cong 1$ s the EP boundary covers about 1.4 km in all directions. But if it continues to move forward along the field, then when moving in transverse (radial) direction it begins to oscillate due to magnetic pressure, resulting in waves that converge

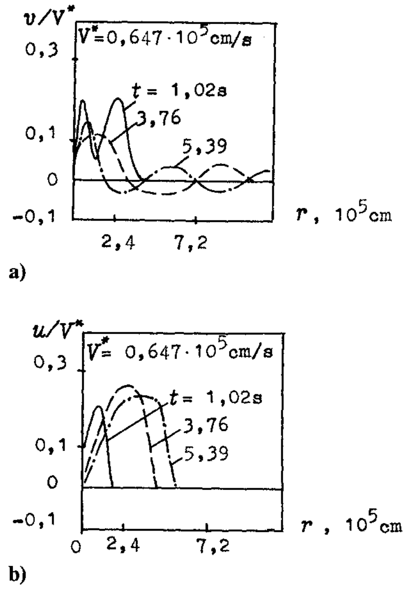

Fig. 5 Same version as Fig. 4: a) radial velocity distribution along the radius in the diametrical plane passing through the center, and b) axial velocity distribution along the axis. The velocities are referred to the sound velocity a = 0.84 km/s.

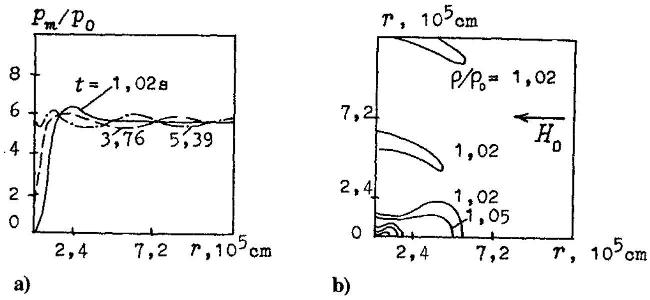

Fig. 6 Same version as Fig. 4: a) magnetic to atmospheric pressure ratio distribution along the radius in the diametrical plane passing through the center, and b) constant density lines in the plane passing through the axis at the time t = 5.39 s (3).

to the axis. Similar oscillations were observed when computing the case of two-dimensional plasma expansion into vacuum.[11] But in the present case the oscillating EP boundary generates a sequence of magnetoacoustic waves propagating the ambient atmosphere across H_0. Their velocities are typically $V_m \cong 2.4$ km/s, and ρ and H amplitudes are about 1–5% (up to times of about 5 s).

The computed velocity V_m agrees well with the theoretical group velocity of the wave obtained from the linearized MHD equations

$$V_m = \left(H_0^2/4\pi\rho_0 + \gamma p_0/\rho_0\right)^{\frac{1}{2}} \cong 2.3 \text{ km/s} \qquad (\gamma = 5/3)$$

Along the magnetic field the disturbance front propagates at a velocity of 0.6 km/s, which is close to the sound velocity. It agrees well with computations without a magnetic field[3] for this experiment.

Three-Dimensional Magnetohydrodynamic Model

The three-dimensional MHD model describes correctly an explosion in a nonuniform atmosphere for arbitrary oriented initial magnetic field H_0 (from the geometric point of view). The flow is symmetrical with respect to the vertical plane in which the magnetic vector H_0 lies. It is reasonable to write the equations in Cartesian coordinates in the form

$$\frac{\partial F}{\partial t} + \frac{\partial P_{ik}}{\partial x_k} + Q = 0, \qquad x_k = x, y, z; \qquad k = 1, 2, 3 \tag{14}$$

Here F is used for the density (ρ), momentum density components (ρu_i), total energy density $[p/(\gamma - 1) + \rho u^2/2 + H^2/8\pi]$, and

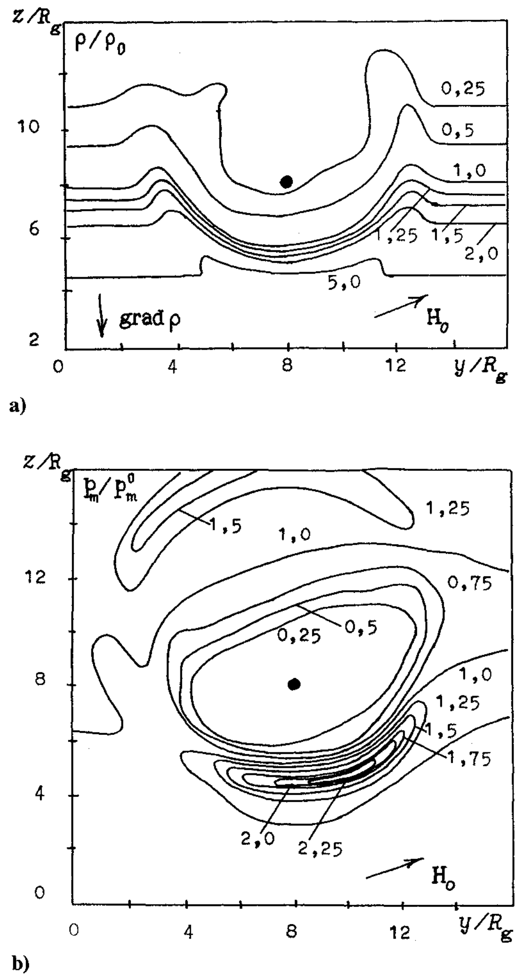

Fig. 7 Explosion with the energy $E = 4 \times 10^{19}$ erg exponential atmosphere at 300-km altitude computed using the three-dimensional MHD-model: a) constant density lines in the plane passing through the vertical (z axis) and the undisturbed magnetic field direction (H_0), density normalized by the atmospheric density at the explosion altitude, $\rho_0 = 2.87 \times 10^{-14}$ g/cm^3, coordinate distance to the hydrodynamic retardation radius $R_g = 20$ km, explosion center marked by the point; and b) constant magnetic pressure normalized by the undisturbed pressure $p_m = 10^{-2}$ dyne/cm^2.

the field component H_i as well. The quantities P_{ik} for values of F that are of hydrodynamic nature are appropriate flow densities: for $F - H_i$ the quantities P_{ik} follow from the field equation. The summand Q describes the gravity-force action, which is introduced to prevent the undisturbed nonuniform atmosphere motion when computing the problem.

The divergence form of the equations provides conservation of the total gas energy and magnetic field when writing the equations in the finite-difference form. However, in the regions where the gas density is extremely small, the magnetic energy can essentially exceed the material energy. In this case small errors in computing H can lead to great errors in the gasdynamic parameters and to the flow pattern distortion. To avoid such errors, a nonconservative form of the finite-difference equations was also used. In this case, the magnetic component is singled out from the energy equation (14) and $p/(\gamma - 1) + \rho u^2/2$ was used as one of the F values.

In general, when computing three-dimensional problems we are faced with computational difficulties, and so it is important to find economic computational schemes. Two explicit economic computational methods with the Euler nets were used, the MacCormack method of second-order accuracy with flow correction and the method of first-second-order accuracy with respect to space variables and of first-order accuracy with respect to time. The second method appears to be more efficient than the first one, therefore, we prefer to use it in serial computations.

The finite-difference schemes, methodological questions, the choice of the effective boundary conditions, variable steps, artificial viscosity or, more briefly, the optimization of the computational process in application to the problems in question are considered in Ref. 15.

The numerical results are illustrated by the example of the explosion of energy $E = 4 \times 10^{19}$ erg, as in the Argus experiment, but at lower altitude $h = 300$ km that is suitable for use of the MHD equations and the atmosphere model (12) (Fig. 7). At an explosion altitude, the initial density is $\rho_0 = 2.9 \times 10^{-14}$ g/cm^3, $p_0 = 1.3 \times 10^{-4}$ dyne/cm^2, the atmospheric pressure is almost 80 times less than the magnetic one, the magnetic retardation radius $R_m \simeq 100$ km is five times greater than that of gasdynamics retardation, $R_g \simeq 20$ km (EP mass $M = 10^6$ g), and the air molecule mean free path is $\ell \simeq 2$ km $\ll R_g, R_m$. Thus, the collisions are frequent, the amount of the air mass involved in the motion is great, and the situation is strongly affected by the atmospheric nonuniformity and magnetic pressure.

The expanding uniform (as to density) plasma ball of radius $R_0 = 10$ km and of kinetic energy E is taken to be an initial condition (corresponding to the initial EP density equal to 8.4 ρ_0).

When expanding and retarding, the major portion of the EP mass first forms a shell near the plasma ambient air interface, and then the mass begins to move in the direction to the center. In the nonsymmetrically converging gas flow, instabilities can arise. The magnetic cavity is gradually deformed, stretching along H_0. The treatment of three-dimensional explosion evolution was restricted in time by the memory size and the efficiency of the computers available, though all of the necessary steps are developed to continue computations for later stages of the expansion which are undoubtedly of interest.

In some versions of the three-dimensional computations, when the initial conditions are nonsymmetric, we observed the appearance and development of MHD instabilities, but this is a special problem that deserves further study.

Conclusion

The computations of high-altitude explosions, taking into account the nonuniform atmosphere and geomagnetic field, give general ideas on the blast wave propagation in the upper atmosphere. The results can be of importance when considering the kinetics of gas ionization, evolution of electron density, and radio frequency wave propagation. They can be used when determining the communication link disruption and magnetic disturbance. In the next paper, the possibility of using the MHD models will be substantiated even for the cases of extremely rarefied media. The approximate validity of the gasdynamical treatment is provided by the small ion Larmor radius in comparison with a typical flowfield scale.

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